

## ESD ACCESSION LIST

ESTI Call No. 65882  
Copy No.     of     cys.

## ESD RECORD COPY

RETURN TO  
SCIENTIFIC & TECHNICAL INFORMATION DIVISION  
(ESTI), BUILDING 1211

Technical Note

1969-31

The Mode Spectrum  
of Avalanche Diodes

H. Berger

3 June 1969

Prepared under Electronic Systems Division Contract AF 19(628)-5167 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts



AD0696996



The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the U.S. Air Force under Contract AF 19(628)-5167.

This report may be reproduced to satisfy needs of U.S. Government agencies.

This document has been approved for public release and sale; its distribution is unlimited.



MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

THE MODE SPECTRUM OF AVALANCHE DIODES

*HENRY BERGER*

*Group 46*

TECHNICAL NOTE 1969-31

3 JUNE 1969

This document has been approved for public release and sale;  
its distribution is unlimited.

## ABSTRACT

Prior studies of IMPATT diodes have restricted their consideration to one-dimensional representations of these devices. In the present report, some qualitatively new modes of behavior are brought to light from a study of a more realistic multidimensional structure. The primary result of this study is the uncovering of a varied spectrum of new avalanche modes, whose character is discussed in some detail.

This spectrum is found to be comprised of two different types of radial TM modes which are referred to as the symmetric and the unsymmetric modes. The totality of prior results of one-dimensional studies turn out to constitute the lowest order mode of the symmetric mode group. This is a quasi-TEM radial wave avalanche mode, which is a bulk mode. The remaining symmetric modes that have been investigated turn out to constitute a new type of avalanche phenomena involving avalanche surface wave modes. The unsymmetric modes correspond most simply to bulk wave modes, but differ considerably from the quasi-TEM bulk mode mentioned above.

Accepted for the Air Force  
Franklin C. Hudson  
Chief, Lincoln Laboratory Office

## CONTENTS

Abstract	iii
I. Introduction	1
II. Fields and Currents	2
III. Dispersion Relations	3
IV. Surface Wave Modes	4
V. Bulk Modes	5
VI. Conclusion	6
Appendix – Device Activity	9

# THE MODE SPECTRUM OF AVALANCHE DIODES

## I. INTRODUCTION

Recent experimental and theoretical investigations have revealed the existence of two-dimensional effects in certain Gunn effect<sup>1,2</sup> and other semiconductor structures.<sup>3,4</sup> These effects usually involve radial variations of the RF fields in the semiconductor, in addition to the expected axial variations parallel to the applied DC electric field. In this report, new microwave frequency modes, with radial variations, are reported for IMPATT diodes biased by an axial DC electric field.

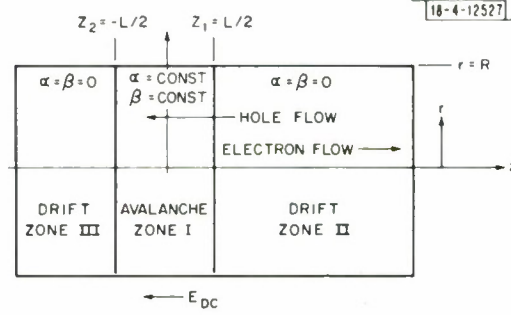


Fig. 1. Multiple uniform layer model of avalanche diode. In the present work, the axial lengths of the drift zones are assumed to be negligible.

The device characteristics for a uniform, right-circular, cylindrical, semiconductor pill of axial length  $L$  and radius  $R$  (see Fig. 1) can be inferred from the avalanche phenomena described by Eqs. (1) and (2) for electron and hole current continuity, and by the Maxwell field equations, Eqs. (3) and (4).

$$\frac{\partial n}{\partial t} = -(1/2) \bar{\nabla} \cdot \bar{J}_n + g \quad , \quad (1)$$

$$\frac{\partial p}{\partial t} = (1/2) \bar{\nabla} \cdot \bar{J}_p + g \quad , \quad (2)$$

$$\bar{\nabla} \times \bar{E} = \mu_0 \frac{\partial \bar{H}}{\partial t} \quad , \quad (3)$$

$$\bar{\nabla} \times \bar{H} = \bar{J}_n + \bar{J}_p + \epsilon \frac{\partial \bar{E}}{\partial t} \quad . \quad (4)$$

Here  $n$  and  $p$  are the electron and hole densities,  $\bar{J}_p = +qp\bar{v}$  and  $\bar{J}_n = -qn\bar{v}$  are the hole and electron current densities which are assumed to flow at the same saturated drift speed  $|\bar{v}|$  parallel to the axial direction,  $\mu_0$  and  $\epsilon$  are the permeability and permittivity of the material,  $\bar{H}$  is the magnetic field,  $\bar{E}$  is the electric field,  $\bar{\nabla}$  is the del operator,  $g = \alpha |\bar{v}|(n + p)$  is electron-hole generation rate, and  $\alpha$  is the ionization rate which, for simplicity, has been assumed to be equal for holes and electrons as in Refs. 5 and 6.

To simplify matters, analysis is restricted to small-signal, time-harmonic fields in the single layer case of distributed avalanche, as in Ref. 5, with  $J_n = 0$  at  $z = L/2$ , and  $J_p = 0$  at  $z = -L/2$ . The influence of the highly conductive regions sandwiching the avalanching space-charge layer is approximated by using the boundary conditions appropriate to infinitely conductive material, i.e., the radial component of  $E$  is zero at  $z = \pm L/2$ .

## II. FIELDS AND CURRENTS

It can be shown that the possible field modes are similar in some respects to the TE and TM radial waveguide modes.<sup>7</sup> When the circumferential variation of the fields and currents is assumed to be negligible (which is consistent with the usual device symmetry), these mode types are uncoupled from each other. In addition, the TE radial modes are uncoupled to the avalanche currents and, hence, are the same as for a "cold" dielectric cylinder bounded by metal caps. Thus, primary interest is in the TM radial modes for which the basic form for the axial component of  $E$  is<sup>7</sup>

$$E_z = AJ_0(Tr) \left\{ e^{-jK_1 z} + e^{+jK_1 z} + B \left[ e^{-jK_3 z} + e^{+jK_3 z} \right] \right\} \quad (5)$$

Here  $J_0(x)$  is the zero<sup>th</sup> order Bessel function of the first kind,  $T$  is the radial wavenumber,  $K_1$  and  $K_3$  are the axial wavenumbers, and  $A$  and  $B$  are constants. Solution of the boundary value problem, posed above, yields a discrete spectrum for  $T$ ,  $K_1$ , and  $K_3$  which in turn implies the same for  $B$ . Each set of associated values for  $T$ ,  $K_1$ , and  $K_3$  corresponds to a particular field and current distribution (which varies with frequency) within the diode that is referred to, for simplicity, as an avalanche mode. Mode is understood here to have the same meaning as, for example, used for empty radial waveguides. The relative amplitudes of the avalanche modes would be determined by imposing boundary conditions at the "rim" of the semiconducting pill, i.e., at  $r = R$ , in the same manner as is done for radial waveguides.

The TM fields and currents, in the avalanche layer under consideration, have been shown to be<sup>7</sup>

$$E_x = J_0(Tr) 2A \{ \cos(K_1 z) + B \cos(K_3 z) \} \quad (6)$$

$$H_\phi = 2j\omega\epsilon TAJ_1(Tr) \left\{ \frac{\cos(K_1 z)}{K_0^2 - K_1^2} + \frac{B \cos(K_3 z)}{K_0^2 - K_3^2} \right\} \quad (7)$$

$$E_r = -2JTAJ_1(Tr) \left\{ \frac{K_1 \sin(K_1 z)}{K_0^2 - K_1^2} + \frac{BK_3 \sin(K_3 z)}{K_0^2 - K_3^2} \right\} \quad (8)$$

$$J_n = J_0(Tr) A \left\{ \sigma_1 \cos(K_1 z) - \frac{jVK_1\sigma_1}{\omega} \sin(K_1 z) + B \left[ \sigma_3 \cos(K_3 z) - \frac{jVK_3\sigma_3}{\omega} \sin(K_3 z) \right] \right\} \quad (9)$$

$$J_p = J_0(Tr) A \left\{ \sigma_1 \cos(K_1 z) + \frac{jVK_1\sigma_1}{\omega} \sin(K_1 z) + B \left[ \sigma_3 \cos(K_3 z) + \frac{jVK_3\sigma_3}{\omega} \sin(K_3 z) \right] \right\} \quad (10)$$

From the boundary condition  $J_p = 0$  at  $z = L/2$  (which is equivalent to  $J_n = 0$  at  $z = -L/2$  because of the symmetry in the present case), Eq. (10) yields

$$B = - \left[ \frac{\omega \cos\theta_1 + jVK_1 \sin\theta_1}{\omega \cos\theta_3 + jVK_3 \sin\theta_3} \right] \left[ \frac{\sigma_1}{\sigma_3} \right] \quad (11)$$

where

$$\sigma_i \equiv \frac{-2j\omega V \alpha'_0 I_0}{\omega^2 + 2j\omega \alpha_0 V - (K_i V)^2} \quad (i = 1, 3), \quad (12)$$

$$\theta_1 \equiv K_1 L/2, \quad \theta_3 \equiv K_3 L/2,$$

$\alpha_0$  = ionization coefficient evaluated at  $E$  equal to the DC bias value,  $E_0$ ,

$\alpha'_0 = d\alpha/dE$  evaluated at  $E = E_0$ ,

$\omega$  = radian frequency,

$I_0$  = DC current density due to  $E_0$ .

### III. DISPERSION RELATIONS

The dispersion relation for the model waves in the avalanche diode can be obtained for Eqs. (8) and (11) and the boundary condition  $E_r = 0$ , at  $z = \pm L/2$ . The resulting dispersion relation is obtained in the simultaneous system of equations:

$$F(\theta_1) = F(\theta_3) \quad , \quad (13)$$

where

$$F(\theta) \equiv \frac{\theta \sin \theta}{\theta_0^2 - \theta^2} \left\{ \frac{1 - [\theta/\theta_d]^2 + j/\theta_d}{\cos \theta + [j\theta \sin \theta]/\theta_d} \right\},$$

$$\theta_0 \equiv \omega \sqrt{\mu_0 \epsilon} L/2, \quad \theta_d \equiv \omega L/2V,$$

and

$$(\theta_{1,3})^2 \equiv \frac{b}{2} \pm \frac{1}{2} \sqrt{b^2 - 4c} \quad , \quad (14)$$

where

$$b \equiv \theta_m^2 + \theta_0^2 - \theta_T^2, \quad \theta_T \equiv TL/2,$$

$$c \equiv \theta_0^2 \theta_m^2 - \theta_T^2 [\theta_m^2 + \theta_a^2],$$

$$\theta_m^2 \equiv \theta_d^2 - \theta_a^2 + j\theta_d,$$

$$\theta_a^2 \equiv [L/2V]^2 2\alpha'_0 I_0 V/\epsilon.$$

There turn out to be two kinds of solutions to the above set of equations. The first kind, which shall be called symmetric for reasons to be discussed shortly, is given approximately by

$$T_n \cong j2n\pi/L \quad , \quad (15)$$

and Eqs. (12) with Eq. (14) used with  $\theta_T^2$  replaced by  $-n^2\pi^2$ . These analytical results have been checked with computer calculations for the modal index number  $n = 0, \pm 1, \pm 2, \pm 3$  and were found



to be accurate to within 5 percent for the usual value of device parameters within the microwave range. It is not clear at present how accurate these results are for  $n > \pm 4$ . However, it can be shown that they definitely fail as  $n$  goes to infinity as follows. From Eq. (13), as  $\Theta_T \rightarrow \infty$ ,

$$\Theta_1^2 \rightarrow 2b \rightarrow -\Theta_T^2 \rightarrow -\infty ,$$

$$\Theta_3^2 \rightarrow c/2b \rightarrow j\Theta_d .$$

Substituting these results into Eq. (13), a contradiction is obtained because

$$1/j\Theta_d \leftarrow F(\Theta_1) \neq F(\Theta_3) \rightarrow [1 - \sqrt{j\Theta_d} \cot(\sqrt{j\Theta_d})]^{-1} .$$

An important simplification occurs in the  $n \leq 3$  cases if  $|\Theta_m|^2 \gg \Theta_d$ , which is often valid in practice. Then,

$$K_{1n}^2 \approx + \frac{4n^2 \pi^2 (\Theta_d^2 + j\Theta_d)}{(\Theta_m^2 + n^2 \pi^2) L^2} , \quad (16)$$

$$K_{3n}^2 \approx K_m^2 + 4n^2 \pi^2 / L^2 - \frac{4n^2 \pi^2 (\Theta_d^2 + j\Theta_d)}{(\Theta_m^2 + n^2 \pi^2) L^2} . \quad (17)$$

#### IV. SURFACE WAVE MODES

The following may be observed for the approximate dispersion relations given by Eqs. (15), (16), and (17):

a. When  $n = 0$ , then

$$T_0 \approx 0 , \quad (18)$$

$$K_{10} \approx 0 , \quad (19)$$

$$K_{30} \approx K_m . \quad (20)$$

This mode very closely approximates a TEM radial wave avalanche mode and will be referred to the quasi-TEM mode.  $T_0$  may be estimated more accurately in the present case by the result for an exact TEM radial wave avalanche mode (which cannot exactly satisfy the boundary conditions but does satisfy them to a good approximation),<sup>7</sup>

$$T_0^2 = -j\omega\mu [\sigma_0 + j\omega\epsilon] , \quad (21)$$

where  $\sigma_0 = \sigma_1(K_1 = 0)$ . The results of one-dimensional theory<sup>5</sup> turn out to constitute this quasi-TEM mode. It should be noted that this mode is a bulk avalanche mode as distinguished from surface wave avalanche modes which are discussed next.

b. The first higher order mode is  $n = \pm 1$  for which

$$T_1^2 \cong -(2\pi/L)^2 , \quad (22)$$

$$K_{11}^2 \cong \frac{4\pi^2(\theta_d^2 + j\theta_d)}{(\theta_m^2 + \pi^2)L^2} \quad (23)$$

$$K_{31}^2 \cong K_m^2 + 4\pi^2/L^2 - 16\pi^2 \frac{(\theta_d + j\theta_d)}{(\theta_m^2 + 4\pi^2)L^2} \quad (24)$$

The qualitative aspects of the behavior for the higher order modes, with  $n = \pm 2, \pm 3$ , are the same as the  $n = \pm 1$  mode. Here the radial wavenumber ( $T_1$ ) is of the same order of magnitude as  $K_{31}$  which for typical device structures is of the order of  $10^4 \text{ cm}^{-1}$ . In addition,  $K_1$  is no longer close to zero (since it involves a factor of  $L^{-1}$ ), although it remains smaller than  $K_{31}$ , and satisfies the simple relation

$$K_{31}^2 + [2K_{11}]^2 \cong K_m^2 + [2\pi/L]^2 \quad (25)$$

c. The total current, defined as  $J_T \equiv J_n + J_p + j\omega\epsilon E$ , varies rapidly with axial position,  $z$ , within the sample length,  $L$ . This is in contrast to the  $n = 0$  case for which  $J_T$  is essentially independent of  $z$ .

d. Because of the rapid decrease of field strength and current with radial distance, these quantities at a distance of  $L$  are substantially less than those at the surface of an avalanching semiconductor pill. Thus, it is appropriate to characterize the  $n \geq 1$  avalanche modes as surface modes.

## V. BULK MODES

In this section, the second category of solutions to the dispersion relations, Eqs. (12) and (14), are discussed. These will be referred to as unsymmetric modes.

The unsymmetric modes are obtained from the dispersion relation solution

$$\theta_1 = \theta_3 \quad (26)$$

and can be determined in closed analytical form. The status of these solutions is not completely clear because  $\theta_1 = \theta_3$  implies  $K_1 = K_3$ . This constitutes a degenerate case for which the derivation of the dispersion relations is not rigorously valid. Those modes are of physical interest because they represent plausible solutions for the unsymmetric case where the saturated electron speed  $V_n$  does not equal the saturated hole speed  $V_p$  (and/or the ionization coefficient for the electron,  $\alpha_n$ , does not equal the ionization coefficient for the holes,  $\alpha_p$ ) in the limit as  $V_n - V_p$  and  $\alpha_n - \alpha_p$  approach zero.

The solutions are plausible because of the continuity normally associated with physical problems, and thus anticipated in the present one. However, a formal analysis of the unsymmetric case (i.e., where  $V_n \neq V_p$ , and/or  $\alpha_n \neq \alpha_p$ ) is required for rigorous justification. In present day devices  $[V_n - V_p]/[V_n + V_p]$  is small but not zero (typically  $0.002 \rightarrow 0.004$ ), and  $[\alpha_n - \alpha_p]/[\alpha_n + \alpha_p]$  is small for Ge and GaAs devices. Thus, these unsymmetric modes may be of considerable physical interest.

The exact unsymmetric modal solutions are determined from Eq. (14) by setting  $b^2 - 4C$  equal to zero. These solutions are



$$T_{1,2,3,4} = \pm jK_a \left\{ 1 \pm \sqrt{1 + [K_m^2 - K_o^2]/K_a^2} \right\} ,$$

$$K_1 = K_3 = 2^{-\frac{1}{2}} \sqrt{K_m^2 + K_a^2 + K_o^2 \pm K_a \sqrt{K_a^2 + K_m^2 - K_o^2}} . \quad (27)$$

Numerical evaluations of the above modal solutions possess radial variations that are relatively moderate, compared to the previously mentioned surface-wave avalanche modes, and thus may be considered a form of bulk-mode waves (see Figs. 2 and 3). However, these radial variations are not negligible, as they are in the more familiar quasi-TEM mode. These unsymmetric modes correspond more closely to the familiar bulk modes for frequencies in the lower portion of the microwave spectrum.

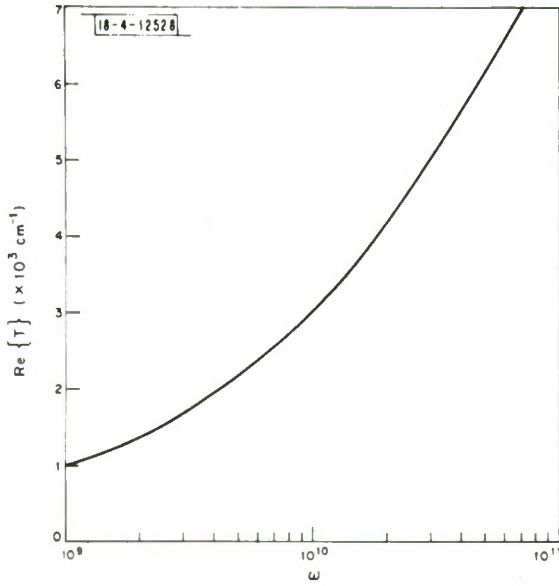


Fig. 2. Real part of the radial wavenumber  $\text{Re}\{T\}$  vs radian frequency  $\omega$ .

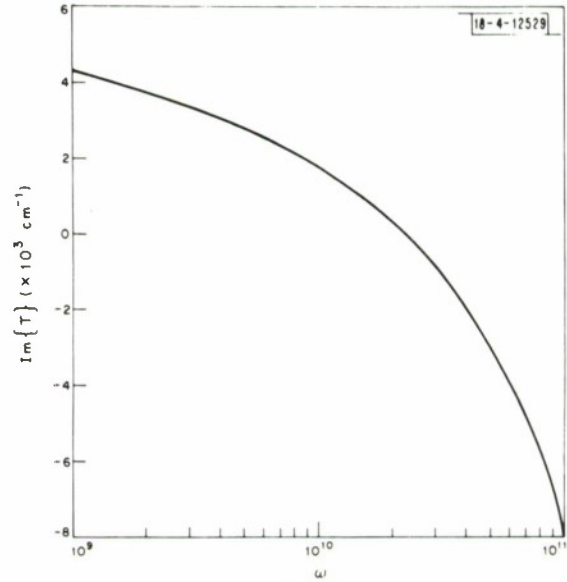


Fig. 3. Imaginary part of the radial wavenumber  $\text{Im}\{T\}$  vs radian frequency  $\omega$ .

The magnetic fields can be determined from the description of  $E$ , for all the modes discussed, from Eq. (3). From this information the impedance of an IMPATT diode, operating in any mode or combination of modes, can be determined in a manner which allows for the finite radius of the diode. This is discussed further in the Appendix.

## VI. CONCLUSION

A variety of new modes of behavior have been uncovered from a detailed examination of a multidimensional avalanche diode structure. The totality of results from prior one-dimensional studies constitute only the lowest order mode of one mode group. The remaining modes exhibit qualitatively different characteristics from the one-dimensional like mode.

The potential existence of these higher order new modes might be anticipated from the observation that the wavelength of the fields in the prior one-dimensional studies is of the same order of size as the axial dimension of the avalanche zone (see Fig. 4). However, it may be initially

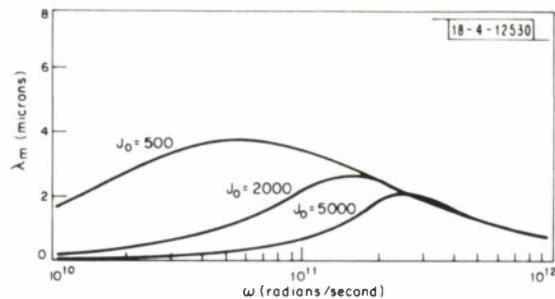


Fig. 4. Axial wavelength of fields in a 1-micron avalanche zone in an avalanche diode determined from  $\lambda_m \equiv \text{Re} \{2\pi/K_m\}$ .

surprising that the radial variations for these new modes are so rapid. Because the electron and hole currents are assumed to be solely in the axial direction (and interact strongly with the axial component of electric field), it is not unexpected that there are longitudinal variations of fields and currents very different from those in the comparable cold dielectric. On the other hand, it might be guessed that the radial variations of fields and currents should approximate those found in the cold dielectric case. The fact that the radial variations are in fact rapid over distances of only several microns is not peculiar to the avalanche effect and arises because the modes are TM and not TEM. Waves with comparable rapid radial variations have been very recently discussed by Kino and Robson for Gunn effect devices,<sup>1</sup> and by Burke and Kino for InSb in the presence of transverse magnetic fields.<sup>3</sup>

The origin of the rapid radial variations for the radial TM avalanche modes can be mathematically traced to the dispersion relation written simply as

$$T^2 = [K^2 - K_0^2] [1 - f(K)] \quad (28)$$

For the quasi-TEM mode,  $f(K)$  is practically one and hence  $T$  is usually negligible. For the TM modes,  $f(K)$  differs considerably from unity and  $T$  varies essentially as  $K \sqrt{1 - f(K)}$ .

For conventional IMPATT diodes (as opposed to sheet or doughnut diodes), one may anticipate that in conventional applications the quasi-TEM bulk mode will dominate all the surface wave modes in device performance. Recent small-signal measurements agree with this assumption and indicate that either the unsymmetric modes are not valid (for reasons discussed earlier) or do not couple well to the microwave cavity modes.

#### ACKNOWLEDGMENT

This report represents an extension of work initiated by the author at The Bayside Research Laboratory of General Telephone and Electronics, Inc. The work was carried out in conjunction with the efforts of Richard I. Harrison of New York University, New York, and Stephen P. Denker of Schlumbere-Doll Research Center, Danbury, Connecticut.



## REFERENCES

1. G. S. Kino and P. N. Robson, "The Effect of Small Transverse Dimensions on the Operation of Gunn Devices," Proc. IEEE 56, 2056-7 (1968).
2. K. Kumabe, "Suppression of Gunn Oscillations by a 2-Dimensional Effect," Proc. IEEE 56, 2172-2173 (1968).
3. B. E. Burke and G. S. Kino, "Observation of Growing Carrier Waves in Indium Antimonide," Appl. Phys. Lett. 12, 310-312 (1968).
4. G. S. Kino, "Carrier Waves in Semiconductors," Microwave Laboratory Report 1353, Stanford University, Stanford, California (August 1965).
5. T. Misawa, "Negative Resistance in p-n Junctions Under Avalanche Breakdown Conditions, Part I," IEEE Trans. ED-13, 137-143 (1966).
6. T. Misawa, "Multiple Uniform Layer Approximation in Analysis of Negative Resistance in p-n Junctions in Breakdown," IEEE Trans. ED-14, 795-808 (1967).
7. H. Berger, R. I. Harrison and S. P. Denker, "Multiple Modes in Avalanche Diodes," to be published.

APPENDIX  
DEVICE ACTIVITY

In general, for any device the electromagnetic power delivered to its environment is given by

$$-P = \text{Re} \left\{ \iiint \frac{\bar{\mathbf{E}} \cdot \bar{\mathbf{J}}^*}{2} dv \right\} \quad (29)$$

for time-harmonic, single-frequency fields, where the integration is over the volume of the device. For the IMPATT device model considered in this study,  $\bar{\mathbf{J}}$  has only a z-component so that

$$-P = \text{Re} \left\{ \iiint \frac{E_z J_z^*}{2} r d\phi dr dz \right\} , \quad (30)$$

where  $r d\phi dr dz$  is the element of volume in cylindrical coordinates.

For the unsymmetric bulk IMPATT modes

$$-P \cong A \text{Re} \left\{ \int_{-L/2}^{+L/2} \frac{E_z J_z^*}{2} dz \right\} , \quad (31)$$

where  $A = \pi R^2$  and the equality is approximate because the radial variations are neglected. It is useful to introduce here the z-component of the total RF current density,  $J_T = J_n + J_p + j\omega\epsilon E_z$ . Then,

$$-P \cong A \text{Re} \left\{ \int_{-L/2}^{+L/2} \frac{E_z J_T^*}{2} dz \right\} . \quad (32)$$

It is further useful to observe that taking the divergence of Eq. (4) yields

$$\bar{\nabla} \cdot \bar{\mathbf{J}}_T = \partial J_T / \partial z + \partial J_r / \partial r = 0 , \quad (33)$$

where  $J_r$  is the radial component of the total RF current density vector  $\bar{\mathbf{J}}_T$ . Since there are no radial conduction currents in the assumed model,  $J_r = j\omega\epsilon E_r$ , so that

$$\frac{\partial J_T}{\partial z} = -j\omega\epsilon \frac{\partial E_r}{\partial r} . \quad (34)$$

It follows from Eqs. (3), (4), and (5) that the radial dependence of  $E_r$  is  $J_1(Tr)$  where  $J_1(x)$  is the first-order Bessel function of the first kind. For moderate radial variations,  $|T|r$  is small compared to unity when  $r$  is not "too large." For that case,  $J_1(Tr) \cong Tr/2$  and hence  $\partial E_r / \partial r$  is independent of  $r$ . Thus, Eq. (34) may be written as

$$\frac{\partial J_T}{\partial z} = a(z) , \quad (35)$$

where  $a(z)$  is independent of  $r$ . A partial integration of Eq. (35) yields

$$J_T = b(z) + g(r) , \quad (36)$$



where  $b(z) = \int a(z) dz$  and  $g(r)$  is independent of  $z$ . On the other hand, it follows from the definition of  $J_T$  and Eqs. (6) through (10) that for any mode  $J_T$  is a product of  $J_0(Tr)$  and some function of  $z$ . The only way these two results for the form of  $J_T$  can be consistent is if

$$J_T = \text{constant} \quad , \quad (37)$$

where the constant is independent of  $r$  and  $z$ . Hence, Eq. (32) becomes

$$-P \cong \frac{A}{2} |J_T|^2 \text{Re} \{ \tilde{Z} \} \quad , \quad (38)$$

where the impedance is

$$\tilde{Z} = V/J_T \quad (39)$$

and where the voltage  $V$  is given by

$$-V = \int_{-L/2}^{+L/2} E_z dz \quad . \quad (40)$$

The preceding argument demonstrates that, if the radial variations are sufficiently small, the power output from radial TM modes is given by the conventional result for which the conventional derivation implicitly considers only a radial TEM mode. Significant errors will arise, however, where  $|T| r$  begins to be comparable to unity. Since  $E_z$  and  $J_T$  vary as  $J_0(Tr)$ , a correction to Eq. (38) can be computed using the approximation

$$J_0(Tr) \cong 1 + (1/4) (Tr)^2 \quad , \quad (41)$$

when  $|T| r$  is still smaller than unity. A correction factor, due to radial variations, is

$$C = \int_0^R |J_0(Tr)|^2 r dr / \frac{R^2}{2} \quad , \quad (42)$$

where for  $|T| r < 1$ ,

$$C \cong 1 + \left[ \frac{T_r R^2}{4} \right] + \text{higher order terms} \quad . \quad (43)$$

For all microwave frequencies one finds, for the case corresponding to Fig. 2, enormous corrections, even for a diode with an unusually small radius such as  $7 \mu$  which has been used in some experiments. Further numerical calculations reinforce the preceding observations that the conventional approximate result for power and impedance, Eq. (38), is not adequate for the unisymmetric bulk modes.

An accurate calculation of device impedance which takes into account the finite device radius can be based on the accurate calculation of power from Eq. (30). This will be discussed in a later work.

DOCUMENT CONTROL DATA - R&D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1. ORIGINATING ACTIVITY (Corporate author)  Lincoln Laboratory, M.I.T.		2a. REPORT SECURITY CLASSIFICATION Unclassified
		2b. GROUP None
3. REPORT TITLE  The Mode Spectrum of Avalanche Diodes		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Note		
5. AUTHOR(S) (Last name, first name, initial)  Berger, Henry		
6. REPORT DATE 3 June 1969	7a. TOTAL NO. OF PAGES 16	7b. NO. OF REFS 7
8a. CONTRACT OR GRANT NO. AF 19(628)-5167	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Note 1969-31	
b. PROJECT NO. 649L	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) ESD-TR-69-144	
c.		
d.		
10. AVAILABILITY/LIMITATION NOTICES  This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES  None		12. SPONSORING MILITARY ACTIVITY  Air Force Systems Command, USAF
13. ABSTRACT  Prior studies of IMPATT diodes have restricted their consideration to one-dimensional representations of these devices. In the present report, some qualitatively new modes of behavior are brought to light from a study of a more realistic multidimensional structure. The primary result of this study is the uncovering of a varied spectrum of new avalanche modes, whose character is discussed in some detail.  This spectrum is found to be comprised of two different types of radial TM modes which are referred to as the symmetric and the unsymmetric modes. The totality of prior results of one-dimensional studies turn out to constitute the lowest order mode of the symmetric mode group. This is a quasi-TEM radial wave avalanche mode, which is a bulk mode. The remaining symmetric modes that have been investigated turn out to constitute a new type of avalanche phenomena involving avalanche surface wave modes. The unsymmetric modes correspond most simply to bulk wave modes, but differ considerably from the quasi-TEM bulk mode mentioned above.		
14. KEY WORDS  avalanche diodes IMPATT diodes surface waves bulk waves  mode spectrum TM radial wave modes multidimensional effects		